THE LOGIC OF ANAPHORA

Jaroslav Peregrin[*]
www.cuni.cz/~peregrin

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Abstract. The paper addresses foundational questions concerning the dynamic semantics of natural language based on dynamic logic of the Groenendijko-Stokhofian kind. Discussing a series of model calculi of increasing complexity, it shows in detail how the usual semantics of dynamic logic can be seen as emerging from the account for certain inferential patterns of natural language, namely those governing anaphora. In this way, the current 'dynamic turn' of logic is argued to be reasonably seen not as the product of changing the focus of logic from the relation of entailment to „a structure of human cognitive action" (van Benthem), but rather as merely another step in our long-term effort to master more and more inferential patterns.

1. The dynamic turn of logic

Classical logic may be seen as having given a faithful account of a certain core part of language. Some of the logics which go beyond its boundaries have then helped to account for other, more intricate parts. One of those elaborations which have proven to be especially interesting has been modal and intensional logic. The idea behind it is that we should let statements denote subsets of a set of indices rather than simply truth values. (Kripke called such indices *possible worlds*, but this term might be misleading, because it seemingly and unwarrantedly transposes Kripke's originally purely logical achievement into the realm of metaphysics.) The fact is that the step from extensional to intensional logic made by Kripke and other semanticists of modal and intensional logic (notably Montague) can be seen as necessitated by the desire to master the modal aspect of language, primarily especially the particles *necessary* and *possibly*, usually regimented as   and ◊, respectively. (It has turned out that other grammatical constructions of natural language, like tenses, are of an essentially similar character too.) And the basic trick of the step, to say it once more, was the passage from seeing statements as names of truth values to seeing them as names of sets.

At present we are facing the effort to master another important aspect of language, which escapes even the modal and intensional conception of logic, the *anaphoric* aspect[1]. The target particles of this dynamic turn of logic appear to be pronouns and articles. It is now also quite clear what this turn amounts to in general[2]: we should stop seeing statements as names

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[1] A formal account of anaphora has become an important problem for semanticists of natural language especially due to the pioneering work of Hans Kamp resulting in his DRT - see Kamp, 1981, and Kamp & Reyle, 1993. Other approaches to anaphora are due to, e.g., Hintikka and Kulas, 1985, or Heim, 1990; purely logical tools of such an account were presented by Greonendijk & Stokhof, 1991.

[2] See Muskens et al. (1987) or van Benthem (1997).

of sets and start to see them as names of certain transformations of 'contexts' or 'information states'. Thus intensions are being replaced by *context-change potentials*; statements are no longer characterized by the possible worlds in which they are true, but rather by the way in which they alter the information states which constitute the contexts of their utterances.

What I think may still not be quite clear enough is the *purely logical rationale* of this step. The engagement of information states within logic is usually explained and justified by epistemological reasons: we need them, so the story goes, to do justice to how human subjects acquire their knowledge or use their language. (Van Benthem, 1997, p. ix, speaks about „the logical structure of cognitive actions, underlying human reasoning or natural language understanding".) This seems to indicate that logic after the 'dynamic turn' is no longer logic in the good old sense of the word: that it is no longer about the inferential patterns governing our language, but now rather about the ways we exercise the patterns; no longer about truth but rather about the way we find out about truth. I think this can be treacherous (one of the basic points of Frege's jump-start into the modern era of logic was the sharp distinction of matters of truth and consequence, which are independent of how anybody may come to know them, from matters of human subjects' coming to recognize that something is true or that something is the consequence of something else); and therefore I think that we should seek a more conservative substantiation of the turn.

The situation is similar within semantics: there too the switch from 'static' notions of meaning to the notion of meaning as a context-change potential is normally explained in terms of the need to account for „how [meaning] affects the information available to illocutionary agents" (Chierchia, 1994, 141). This again indicates that the 'dynamic turn' alters the subject matter of semantics: while earlier semantics addressed what expressions meant (perhaps their truth conditions), now it is to address (also) how they change the context. The turn thus might seem to amount to simply a deliberate shift of the boundaries of semantics, which made it include also some parts of what has been previously considered a matter of, say, the theory of communication. Again, I think this may be misleading, for the engagement of context-change potentials need not be the result of moving the boundaries of semantics, but merely of paying due attention to some of those expressions and locutions of natural language which were ignored before (pronouns, articles, anaphoric locutions). Thus also here a more conservative substantiation might be in place.

What I mean by 'a more conservative substantiation' can be elucidated by volunteering a parallel between the dynamic turn with its *information states* and the intensional turn with its *possible worlds*. We can, of course, give various epistemological (or even psychological) reasons for the employment of intensional logic and possible worlds; but we can justify it also purely logically. The fact is that we need the operator   as governed by certain axioms for the purpose of capturing certain inferential patterns which play an important role within our language and thereby within our reasoning. And another fact is that arguably the simplest and most perspicuous model theory for the relevant calculus treats statements as names of elements of the powerset of a fixed set and   as a designation of a mapping of the power-set on itself. From this vantage point, possible-worlds-semantics is the natural outcome of the effort to account for certain inferential patterns of our language. And what I want to do in this paper is to render the information-states-semantics (or the semantics of context-change potentials) as the natural outcome of the effort to account for other kind of inferential patterns; namely those involving anaphora. This is to say that what I want to do is to point out the basic inferential patterns characterizing 'the logic of anaphora' and show how they can be seen to yield the basic framework of dynamic semantics.

2. Anaphora from the Viewpoint of Inference

As the paradigmatic cases of inferences involving anaphora we shall consider (1)-(4):

John walks. He whistles. $\Rightarrow$ *John walks and whistles.* (1)
Somebody walks. He whistles. $\Rightarrow$ *Somebody walks and whistles.* (2)
A man walks and a woman runs. He whistles and she smiles. $\Rightarrow$
A man walks and whistles. A woman runs and smiles. (3)
A man walks. The man whistles $\Rightarrow$ *A man walks and whistles* (4)

When we consider the possibility of capturing these instances within the framework of a predicate-calculus-like logic, we probably come to the conclusion that at least the first three of them could be accommodated quite easily if we managed to extend the calculus by means of pronoun-like terms. Then they may acquire the form of (1')-(3').

$$\mathrm{Wa(J)\ \&\ Wh(he)} \Rightarrow \mathrm{Wa(J)\ \&\ Wh(J)} \quad (1')$$
$$\exists \mathrm{xWa(x)\ \&\ Wh(he)} \Rightarrow \exists \mathrm{x(Wa(x)\ \&\ Wh(x))} \quad (2')$$
$$\exists \mathrm{x(Ma(x)\ \&\ Wa(x))\ \&\ }\exists \mathrm{x(Wo(x)\ \&\ Ru(x))\ \&\ Wh(he)\ \&\ Sm(she)} \Rightarrow$$
$$\exists \mathrm{x(Ma(x)\ \&\ Wa(x)\ \&\ Wh(x))\ \&\ }\exists \mathrm{x(Wo(x)\ \&\ Ru(x)\ \&\ Sm(x))} \quad (3')$$

The question now is *how* we could add terms of this kind to the predicate calculus.

3. The Semantics of 'Backwards-Looking'

Let us start from (1'). As the only types of expressions it contains aside of the problematic *he* are individual constants, unary predicate constants and conjunction, let us, for simplicity's sake, start from a very simple language whose vocabulary is restricted just to these expressions. Hence the following language, which we shall call L.

Expressions of L fall into three categories: individual constants, unary predicate constants and connectives (the last category consisting of the single constant **&**). If p is a predicate constant and i is an individual constant, $p(i)$ is a statement; and if s_1 and s_2 are statements, s_1**&**s_2 is a statement. The standard semantics of L is then as one would expect: If U is a set, then $\|\ \|$ is an interpretation of L in U iff $\|i\| \in$ U for every individual constant i, $\|p\| \subseteq$U for every predicate constant p. The interpretation then extends to an assignment of truth values to statements in the natural way: for every individual constant i and every predicate constant p, $\|p(i)\|$ = T iff $\|i\| \in \|p\|$; and for every statements s_1 and s_2, $\|s_1 \mathbf{\&} s_2\|$ = T iff $\|s_1\|$ = T and $\|s_2\|$ = T. (The exact specifications of all the calculi discussed within the text can be found in the appendix.)

In order to be able to enrich L with a constant behaving like *he* in (1'), we shall consider an alternative semantics for L, which is as follows. Individual constants are taken to denote functions from U to U, each of them being assigned a constant function defined everywhere on U. The denotations of predicate constants are as before. This interpretation induces an assignment to statements of functions from U to U in the following way: If i is an individual constant and p is a predicate constant, then $\|p(i)\|$ is a (partial) function from U to U such that for every x$\in$U, $\|p(i)\|$(x) is defined iff $\|i\|$(x)$\in \|p\|$, and in this case $\|p(i)\|$(x)

$=\|i\|(x)$. (Thus, $\|p(i)\| = \{\langle x,y\rangle \mid \langle x,y\rangle \in \|i\|$ and $y\in\|p\|\}$; and as $\|i\|$ is a constant function, $\|p(i)\|$ is either identical with $\|i\|$, or is empty.) If s_1 and s_2 are statements, then $\|s_1\&s_2\|$ is such function that for every $x\in U$, $\|s_1\&s_2\|(x) = \|s_2\|(\|s_1\|(x))$; i.e. $\|s_1\&s_2\| = \{\langle x,y\rangle \mid$ there is a z such that $\langle x,z\rangle\in\|s_1\|$ and $\langle z,y\rangle\in\|s_2\|\}$. A statement is *true*, by definition, if its denotation (which is a function) is defined everywhere on U; it is *false* if it is defined nowhere on U.

It is easy to see that under any such alternative interpretation, every statement is either true or false. It is also easy to see that for each standard interpretation of L there exists an equivalent alternative interpretation and vice versa. (If $\|\ \|_S$ is a standard interpretation, then the equivalent alternative interpretation $\|\ \|_A$ is defined in the following way: $\|i\|_A = \{\langle x,y\rangle \mid x\in U$ and $y = \|i\|_S\}$ for every individual constant i and $\|p\|_A = \|p\|_S$ for every predicate constant p. If, conversely, $\|\ \|_A$ is an alternative interpretation, then the corresponding $\|\ \|_S$ is defined in such a way that $\|i\|_S$ is the constant value of $\|i\|_A$.)

Now let us enrich L with a 'backward-looking' term $\leftarrow$ having the property that for every individual constant i and every predicate constant p_1 and p_2, $p_1(i)\&p_2(\leftarrow)$ is equivalent to $p_1(i)\&p_2(i)$. We can do this easily if we start from the just defined alternative semantics for L - for then the desired equivalence is secured by taking $\|\leftarrow\|$ to be the identity function defined everywhere on U. To see that this works, let us consider the value of $\|p_1(i)\&p_2(\leftarrow)\|$ applied to an element x of the universe. If we denote the single value of the constant function $\|i\|$ as x_i, then $\|p_1(i)\&p_2(\leftarrow)\|(x) = \|p_2(\leftarrow)\|(\|p_1(i)\|(x))$, where $\|p_1(i)\|$ is either a constant function (if $x_i\in\|p_1\|$) or a function defined nowhere (otherwise), and $\|p_2(\leftarrow)\|$ is an identical function defined for those and only those x for which $x\in\|p_2\|$. Thus, $\|p_2(\leftarrow)\|(\|p_1(i)\|(x))$ is defined (and yields x_i) iff $x_i\in\|p_1\|$ and $x_i\in\|p_2\|$, thus it is defined if and only if $\|p_2(i)\|(\|p_1(i)\|(x))$ is. Hence, $\|p_1(i)\&p_2(\leftarrow)\|$ is true (= defined for every x) if and only if $\|p_1(i)\&p_2(i)\|$ is true (defined for every x).

Let us call this language $L^{\leftarrow}$. $L^{\leftarrow}$ differs from L (considered with the alternative semantics) in that it contains formulas denoting functions which are defined for some elements of the universe and undefined for others, hence formulas which are, according to our definition of truth, neither true, nor false. This is the case of, for instance, any formula $p(\leftarrow)$ with $\|p\|$ being a nonempty, proper subset of U. If we realize that $\leftarrow$ should play the role of a pronoun, then this should not surprise us: if *whistle*($\leftarrow$) is to render *he whistles*, then no wonder than it is, when uttered out of the blue, neither true, nor false.

4. 'Backward-Looking' Terms and Indeterminacy

In this way, we have developed a language which does justice to (1) (our $\leftarrow$ being the regimentation of (1)'s *he*); so let us proceed to (2). To be able to accommodate it, we need existential quantification; however, we shall not introduce it in the usual way. Let us return back to L and let us now consider another kind of alternative semantics for it. Individual constants will now be taken to denote subsets of the universe, each individual constant being assigned a singleton; and if i is an individual constant and p a predicate constant, then $\|p(i)\| = T$ iff $\|i\|\cap\|p\| \neq \emptyset$. Everything else is as in the standard case. That this alternative semantics is equivalent to the standard one is obvious.

Starting from this semantics, we can define the language L^{ε} by adding to L the new term $\boldsymbol{\varepsilon}$ which is taken to denote the whole universe, $\|\boldsymbol{\varepsilon}\| = U$. Then it is easy to see that $p(\boldsymbol{\varepsilon})$ is true if and only if there is something which is *p*, i.e. that what the statement claims is, in traditional notation, $\exists x.p(x)$.

This form of existential quantification is now suitable to interact with the backward-looking term $\leftarrow$. What we need to do is to combine L^{ε} with $L^{\leftarrow}$; so what we first need is to merge the two alternative semantics of L which we have employed to produce $L^{\leftarrow}$ and L^{ε}, respectively. This can be done in a straightforward way: denotations of terms (where terms are now individual constants, $\boldsymbol{\varepsilon}$ and $\leftarrow$) will now be binary relations over U such that for every ordinary individual constant *i*, $\|i\| = \{<x,x_i> \mid x\in U\}$ for some $x_i\in U$, $\|\boldsymbol{\varepsilon}\| = \{<x,y> \mid x\in U \text{ and } y\in U\}$ and $\|\leftarrow\| = \{<x,x> \mid x\in U\}$. The denotation of predicates remains unchanged (i.e. they keep denoting subsets of U). Also statements now denote binary relations over U, their denotations being defined in the following way: $\|p(t)\| = \{<x,y> \mid <x,y>\in \|t\| \text{ and } y\in \|p\|\}$; $\|s_1\&s_2\|=\{<x,y> \mid \text{there is a z such that } <x,z>\in \|s_1\| \text{ and } <z,y>\in \|s_2\|\}$. A statement *s* is *true* iff for every $x\in U$ there exists a $y\in U$ such that $<x,y>\in \|s\|$; it is *false* iff for no $x\in U$ is there a $y\in U$ such that $<x,y>\in \|s\|$ (i.e. iff $\|s\| = \emptyset$).

It is now easy to see that within the resulting language $L^{\varepsilon,\leftarrow}$ not only is $p_1(i)\&p_2(\leftarrow)$ equivalent to $p_1(i)\&p_2(i)$ for every individual constant *i* and every predicate constant p_1 and p_2, but $p_1(\boldsymbol{\varepsilon})\&p_2(\leftarrow)$ is true if and only if there is an x which is both p_1 and p_2. The proof of the former claim is straightforwardly analogous to the proof of the same claim within $L^{\leftarrow}$; so let us prove the latter one only. If *s* is a sentence of $L^{\varepsilon,\leftarrow}$, then we shall write $x\|s\|y$ instead of $<x,y>\in \|s\|$. Then we can see that $x\|p_1(\boldsymbol{\varepsilon})\&p_2(\leftarrow)\|y$ iff there is a z such that $x\|p_1(\boldsymbol{\varepsilon})\|z$ and $z\|p_2(\leftarrow)\|y$. Moreover, $x\|p_1(\boldsymbol{\varepsilon})\|z$ iff $x\|\boldsymbol{\varepsilon}\|z$ and $z\in \|p_1\|$, i.e. iff $z\in \|p_1\|$ (for $x\|\boldsymbol{\varepsilon}\|z$ holds for every x and z); and $z\|p_2(\leftarrow)\|y$ iff $z = y$ and $y\in \|p_2\|$. Hence $x\|p_1(\boldsymbol{\varepsilon})\&p_2(\leftarrow)\|y$ iff $y\in \|p_1\|$ and $y\in \|p_2\|$. This means that $\|p_1(\boldsymbol{\varepsilon})\&p_2(\leftarrow)\|$ is true iff there is a y such that $y\in \|p_1\|$ and $y\in \|p_2\|$ (for then $x\|p_1(\boldsymbol{\varepsilon})\&p_2(\leftarrow)\|y$ for every $x\in U$).

The fact that statements of $L^{\varepsilon,\leftarrow}$ denote relations between individuals of the universe invites a ‘dynamic’ reading: what a statement denotes can be seen as an (indeterministic) transition from an individual to an individual, as something that ‘consumes’ an individual (which is yielded by a previous statement - if any) and ‘produces’ a (possibly different) individual (which is then consumed by a subsequent statement - if any). We may see it also in terms of a ‘saliency box’ the content of which may be supplied by one statement and subsequently utilized by another one: the statement’s input is what is in the box when the statement is uttered (the box may be empty or filled by an individual), and its output is the content of the box as established by the utterance (the box may be unchanged, or (re)filled by a new individual). In such terms we can describe the semantics of $L^{\varepsilon,\leftarrow}$ in an illuminating way: an ordinary constant always (re)fills the saliency box by a fixed individual; $\leftarrow$ leaves the contents of the box unchanged, and $\boldsymbol{\varepsilon}$ fills the box ‘indeterministically’ by an arbitrary individual. A subject-predicate statement then ‘works’ (and thereby is *true*) in a given context if the contents of the saliency box as produced by the subject of the statement (in that context) belongs to the subset of the universe which is denoted by the predicate of the statement. However, it is good to notice that this story is meant neither as a depiction of something going on within speakers’ heads, nor as a description of a structure of ‘cognitive actions’; it is a metaphorical way of envisaging a model theory for a language which we have found to do justice to inferences like (1) and (2).

5. Multiple 'Backward-Looking' Terms

Now let us turn our attention to (3). What we need in this case is a plurality of different 'pronouns' ('backward-looking' terms). To provide for it, let us modify $L^{\varepsilon,\leftarrow}$ in the following way: let us divide the terms of the language into n disjoint sorts, so that every individual constant falls into one of the sorts, and $\boldsymbol{\varepsilon}$ and $\leftarrow$ are replaced by $\boldsymbol{\varepsilon}_k$ and $\leftarrow_k$ for every sort k. $\|\ \|$ is an interpretation of this language iff $\|t\|\in U^n x U^n$ for every term t and $\|p\|\subseteq U$ for every predicate constant p. Let us write $[x]_n$ instead of $<x_1,...,x_n>$ and omit the subscript n where no confusion is likely to arise; and let us write $[x]^i$ for the i-th constituent of the sequence [x]. For every k, $\|t_k\|$ is some set of pairs of n-tuples <[x],[y]> such that for $j\neq k$, $[y]^j=[x]^j$; if the t_k is $\boldsymbol{\varepsilon}_k$ then it is the set of all such pairs, if it is $\leftarrow_k$, then it is the set of all such pairs for which also $[y]^k=[x]^k$, and for an ordinary individual constant it is the set of all such pairs for which $[y]^k$ is a fixed element of the universe. In symbols, let $\|\boldsymbol{\varepsilon}_k\| = \{<[x],[y]> \mid [y]^j=[x]^j \text{ for } j\neq k\}$, $\|\leftarrow_k\| = \{<[x],[y]> \mid [y]=[x]\}$, and for every individual constant i_k of the sort k let there exist an individual x_{ik} so that $\|i_k\| = \{<[x],[y]> \mid [y]^k= x_{ik};\ [y]^j=[x]^j \text{ for } j\neq k\}$.

Such an interpretation now induces an assignment of elements of $U^n x U^n$ to statements in the following way: $\|p(t_k)\| = \{<[x],[y]> \mid <[x],[y]>\in \|t_k\| \text{ and } [y]^k \in \|p\|\}$ if t_k is a term of the sort k, and $\|s_1\&s_2\| = \{<[x],[y]> \mid \text{there is a } [z] \text{ so that } <[x],[z]>\in\|s_1\| \text{ and } <[z],[y]>\in\|s_2\|\}$. A sentence s is *true* iff for every $[x]\in U^n$, there is an $[y]\in U^n$ so that $<[x],[y]>\in\|s\|$; it is *false* if for no $[x]\in U^n$ there is an $[y]\in U^n$ so that $<[x],[y]>\in\|s\|$ (i.e. iff $\|s\|=\emptyset$).

Now let us show that $p_1(i_k)\&p_2(\leftarrow_k)$ is equivalent to $p_1(i_k)\&p_2(i_k)$ for every individual constant i_k of the sort k (and every predicate p_1 and p_2). By definition, $[x]\|p_1(i_k)\&p_2(\leftarrow_k)\|[y]$ iff there is a [z] so that $[x]\|p_1(i_k)\|[z]$ and $[z]\|p_2(\leftarrow_k)\|[y]$. Furthermore, $[x]\|p_1(i_k)\|[z]$ iff $[x]\|i_k\|[z]$ and $[z]^k\in\|p_1\|$; and $[z]\|p_2(\leftarrow_k)\|[y]$ iff $[z]\|\leftarrow_k\|[y]$, and $[y]^k\in\|p_2\|$. Hence $[x]\|p_1(i_k)\&p_2(\leftarrow_k)\|[y]$ iff $[x]\|i_k\|[z]$ and $[z]^k\in\|p_1\|$ and $[z]\|\leftarrow_k\|[y]$, and $[y]^k\in\|p_2\|$. But as $[z]\|\leftarrow_k\|[y]$ iff $[z] = [y]$, this reduces to $[y]^k = x_{ik}$ and $[y]^j=[x]^j$ for $j\neq k$ and $x_{ik}\in\|p_1\|$ and $x_{ik}\in\|p_2\|$, which is clearly also a necessary and sufficient condition for $[x]\|p_1(i_k)\&p_2(i_k)\|[y]$. Thus, $[x]\|p_1(i_k)\&p_2(\leftarrow_k)\|[y]$ if and only if $[x]\|p_1(i_k)\&p_2(i_k)\|[y]$.

In contrast to this, let us show that $p_1(i_k)\&p_2(\leftarrow_r)$ is *not* equivalent to $p_1(i_k)\&p_2(i_k)$ for $k\neq r$. By reasoning analogous to that of the previous paragraph, we see that $[x]\|p_1(i_k)\&p_2(\leftarrow_r)\|[y]$ iff $[x]\|i_k\|[y]$ and $[y]^k\in\|p_1\|$ and $[y]^r\in\|p_2\|$; and as $[x]\|i_k\|[y]$ implies $[x]^r = [y]^r$, this further reduces to $[x]\|i_k\|[y]$ and $[y]^k\in\|p_1\|$, and $[x]^r\in\|p_2\|$. Now let $\|\ \|$ be such an interpretation that the constant value of $\|i_k\|$, x_{ik}, is a member of both $\|p_1\|$ and $\|p_2\|$, but $\|p_2\| \neq U$. Let us take an n-tuple of objects [x] so that $[x]^r\notin\|p_2\|$. Then, as we have just seen, there is no [y] so that $[x]\|p_1(i_k)\&p_2(\leftarrow_r)\|[y]$. However, at the same time it is obvious that there *is* an [y] so that $[x]\|p_1(i_k)\&p_2(i_k)\|[y]$, namely the [y] defined as follows: $[y]^k = x_{ik}$ and $[y]^j = [x]^j$ for $j\neq k$. This means that $p_1(i_k)\&p_2(\leftarrow_r)$ and $p_1(i_k)\&p_2(i_k)$ are *not* equivalent.

Similarly we could show that $p_1(\boldsymbol{\varepsilon}_k)\&p_2(\leftarrow_k)$ is true iff there is something which is both p_1 and p_2, whereas $p_1(\boldsymbol{\varepsilon}_k)\&p_2(\leftarrow_r)$ may be true even if there is no such x (and may be false even if there is).

It would be also possible to introduce constants which are of more sorts than one: if $\|i\| = \{<[x],[y]> \mid [y]^k=[y]^r=c;\ [y]^j=[x]^j \text{ for } k\neq j\neq l\}$, then both $p_1(i)\&p_2(\leftarrow_k)$ and $p_1(i)\&p_2(\leftarrow_r)$ are equivalent to $p_1(i)\&p_2(i)$; but $p_1(i)\&p_2(\leftarrow_s)$ is *not* equivalent to $p_1(i)\&p_2(i)$ for $k\neq s\neq r$.

Returning back to our inferential patterns, we can see that to do justice to (3) we need $L^{\varepsilon,\leftarrow,3}$ with the three sorts rendering natural language grammatical gender (masculina, feminina, neutra). The operators $\leftarrow_1$, $\leftarrow_2$, $\leftarrow_3$ would then correspond to 'he', 'she', 'it', respectively; and the operators $\boldsymbol{\varepsilon}_1$, $\boldsymbol{\varepsilon}_2$, $\boldsymbol{\varepsilon}_3$ would provide a more refined resource for which English only has the two particles 'somebody' and 'something'.

In general, we can see the expressions of $L^{\varepsilon,\leftarrow,n}$ in the following way. We have 'introducers', which are either definite (ordinary individual constants) or indefinite (epsilons), and 'pickers' (backarrows). Introducers interact with pickers to yield the anaphoric structure of the discourse, but not just any introducer with just anypicker; to interact with an introducer, the picker has to be 'tuned to the same frequency' as the introducer. The 'frequency' to which a picker or an introducer is 'tuned' is its sort. We have a finite number of 'frequencies'.

It is even more illuminating to see the situation again in terms of a saliency box, this time with n slots instead of just one, the slots corresponding to that which has been just envisaged as 'frequencies'. The introducers of the sort k fill the k-th slot of the box; the pickers of the sort l pick up the contents of the l-th slot.

6. Complex 'Backward-Looking' Terms

So far, the only 'introducers' and 'pickers' were words (primitive constants). Thus we have developed a framework to account for the inferential patterns of the kind of (3); but what about those of the kind of (4)? Here we have to depart from the structure of the predicate calculus more substantially; but our previous investigations can give us a lead.

It seems that besides the simple ones, in natural language we have also complex 'introducers' and 'pickers': *a man* and *the man* underlying the inferential pattern (4) seem to be an example. Introducers like *a man*, *a logician*, or *a killer*, despite being all tuned to the general 'masculine frequency' (and thus being capable of interacting with *he*), seem to provide also a finer frequency key. They do not interact with the corresponding pickers, *the man*, *the logician*, or *the killer*, indiscriminately: *A logician walks and the killer whistles* does not in general say that there is somebody who walks and whistles[3]. We have to imagine an infinite number of frequencies and a mechanism which turns a general name into either an 'introducer' or a 'picker'. And indeed, this seems to be what we have in English and what underlies the inferential pattern (4): articles. Thus instead of employing the multiple $\boldsymbol{\varepsilon}$'s and $\leftarrow$'s, we can use two constants, say **a** and **the** (to make them wear their function on their sleeves), constituting a new category of expressions applicable to predicates to form terms.

To see what semantics this new calculus, call it $L^{the,a}$, should have, let us return to the image of the saliency box once more. So far, the box consisted of n slots, each of which was suited to contain the current salient item of the sort n. Thus, the slots of the box may be seen as labeled by integers up to n. Now what we obviously need is an infinity of slots; more precisely we need an individual for each semantically distinct predicate. This is to say that the slots of the box are now to be labeled by denotations of predicates, i.e. subsets of the universe; which means that statements of $L^{the,a}$ are to be seen as denoting relations between functions which assign elements of the universe to subsets of the universe. Realizing that it is plausible to work only with such functions which map a set always on an element of itself

[3] Although this might be felt as a kind of a 'conversational implicature'.

('the most salient p' should, of course, be a p), we reach the framework of Peregrin & von Heusinger (1997).

To be more rigorous: the terms of $L^{the,a}$ are individual constants plus complex terms formed by applying **a** and **the** to predicates. Let U be the universe and let CHF_U denote the set of choice functions over U, i.e. the set of all functions C such that (i) the domain of C is included in Pow(U), (ii) the range of C is included in U, and (iii) $C(s) \in s$ for every s from the domain of C. The semantics of $L^{the,a}$ is as follows: each term and each statement is assigned a subset of $CHF_U x CHF_U$; each predicate is assigned a subset of U. The 'value referred to by t_k in the context c', $|t_k|_c$, where the context is identified with a choice function, is defined in the following way: if t_k is $\mathbf{a}(p)$ or $\mathbf{the}(p)$, then $|t_k|_C = C(\|p\|)$, whereas if t_k is an individual constant i_k, then $|t_k|_c = x_{ik}$ for some fixed $x_{ik} \in U$ (independently of C). If p is a predicate and t a term, then $\|p(t)\| = \{<C,C'> \mid <C,C'> \in \|t\|$ and $|t|_{C'} \in \|p\|\}$. If i is an individual constant, then $\|i\| = \{<C,C> \mid C \in CHF_U\}$[4]; if p is a predicate, then $\|\mathbf{a}(p)\| = \{<C,C'> \mid C(s) = C'(s)$ for $s \neq \|p\|$ and $C'(\|p\|) \in \|p\|\}$; and $\|\mathbf{the}(p)\| = \{<C,C> \mid C(\|p\|) \in \|p\|\}$. If we again stipulate that a statement s is *true* (*false*) iff for every (no) $C \in CHF_U$ there exists a $C' \in CHF_U$ so that $<C,C'> \in \|s\|$, then we can easily prove that $p_1(\mathbf{a}(p))\mathbf{\&}p_2(\mathbf{the}(p))$ is true if and only if there is an item which is p, p_1 and p_2; and hence the inferential pattern (4) is validated.

To prove it, let us compute the denotation of $p_1(\mathbf{a}(p))\mathbf{\&}p_2(\mathbf{the}(p))$. By definition, $C\|p_1(\mathbf{a}(p))\mathbf{\&}p_2(\mathbf{the}(p))\|C'$ iff there is a C'' such that $C\|p_1(\mathbf{a}(p))\|C''$ and $C''\|p_2(\mathbf{the}(p))\|C'$. But as $C\|p_1(\mathbf{a}(p))\|C''$ iff $C\|\mathbf{a}(p)\|C''$ and $C''(\|\mathbf{a}(p)\|) \in \|p_1\|$, while $C''\|p_2(\mathbf{the}(p))\|C'$ iff $C''\|\mathbf{the}(p)\|C'$ and $C'(\|\mathbf{the}(p)\|) \in \|p_2\|$, it is obvious that $C\|p_1(\mathbf{a}(p))\mathbf{\&}p_2(\mathbf{the}(p))\|C'$ iff $C\|\mathbf{a}(p)\|C''$ and $C''(\|\mathbf{a}(p)\|) \in \|p_1\|$ and $C''\|\mathbf{the}(p)\|C'$ and $C'(\|\mathbf{the}(p)\|) \in \|p_2\|$. And as $C\|\mathbf{a}(p)\|C''$ iff $C(s) = C'(s)$ for $s \neq \|p\|$ and $C'(\|p\|) \in \|p\|$, while $C''\|\mathbf{the}(p)\|C'$ iff C''=C' and $C''(\|p\|) \in \|p\|$, this further reduces to $C(s) = C'(s)$ for $s \neq \|p\|$ and $C'(\|p\|) \in \|p\|$ and $C'(\|\mathbf{a}(p)\|) \in \|p_1\|$ and $C'\|p\| \in \|p\|$ and $C'(\|\mathbf{the}(p)\|) \in \|p_2\|$, which means that C' differs from C at most in the value it assigns to $\|p\|$ and this value is an element of $\|p\|$, $\|p_1\|$ and $\|p_2\|$. Hence, given C, the existence of C' for which $C\|p_1(\mathbf{a}(p))\mathbf{\&}p_2(\mathbf{the}(p))\|C'$ is tantamount to the existence of something which is p, p_1 and p_2. Q.e.d.

7. De-simplification

The formal languages we have discussed so far are, of course, so simple that they cannot be taken too seriously. The reason for employing such impoverished languages was to make the exposition of the basic logical backbone of the apparatus quite transparent; and now we are going to indicate how we could put flesh back on the bones. We shall talk about $L^{\varepsilon,\leftarrow}$, the cases of the more complex languages are straightforwardly analogous.

First, we have restricted ourselves to unary predicates. However, there is no problem of principle in accepting predicates of greater arities. In fact, there are at least two distinct ways to handle them. Informally speaking, these two ways differ in the order in which we compute the referents of terms of a two- or more-place predicate (which are to be tested

[4] This means that individual constants are taken to trigger no anaphoric effects. This is, of course, a substantial oversimplification. For a more elaborate treatment see Peregrin & von Heusinger (1997).

against the denotation of the predicate): we may compute a term's referent either immediately after *the very term* exercises its context-change potential, or else after *every term* does so. For $L^{\varepsilon,\leftarrow}$, we have the following two alternative rules:

$<x,y> \in \|p(t_1,...,t_n)\|$ iff there are $x_0,...,x_n$ so that $x = x_0$, $y = x_n$, $<x_{j-1},x_j> \in \|t_j\|$ for $j = 1,...,n$, and $<x_1,...,x_n> \in \|p\|$

$<x,y> \in \|p(t_1,...,t_n)\|$ iff there are $x_1,...,x_n$ so that $y = x_n$, $<x,x_j> \in \|t_j\|$ for $j = 1,...,n$, and $<x_1,...,x_n> \in \|p\|$

Which of these options is more adequate from the viewpoint of the language's capacity to adequately reconstruct real inferential patterns of natural language is a matter of careful linguistic analysis which will not be discussed here.

Second: negation, implication and other usual logical operators. The accommodation of these operators within a dynamic logical framework cause problems discussed in detail in the literature. They are again mostly problems of having to choose between various alternatives. The most straightforward way to accommodate the operators within $L^{\varepsilon,\leftarrow}$ seems to be (in analogy with what has been proposed by Greonendijk and Stokhof, 1991):

$\|\neg s\| = \{<x,x> \mid \text{there is no } y \text{ so that } <x,y> \in \|s\|\}$

$\|s_1 \rightarrow s_2\| = \{<x,x> \mid \text{for every } <x,y> \in \|s_1\| \text{ there is a } z \text{ so that } <y,z> \in \|s_2\|\}$

However, the most controversial feature of the approach proposed here is that the languages presented avoided ordinary quantification in favor of indefinite terms with existential import. The reason for this is that this is, I am convinced, the right way to make the structure of the languages close enough to that of natural language to be able to account for the inferential patterns involving anaphora. However, if we want to make the present model into more than a toy, we undoubtedly have to indicate how it could be enriched with something approaching the power of quantifiers of ordinary logic.

$L^{\varepsilon,\leftarrow}$ obviously does not enable us even to spell out that there is something which is p_1 and p_2. The formula $p_1(\boldsymbol{\varepsilon})\&p_2(\boldsymbol{\varepsilon})$ says that there is something which is p_1 and something which is p_2, not that there is necessarily something with both the properties. How could we do away with this restriction? Remember that we invented $\boldsymbol{\varepsilon}$ because this seemed to be more congenial to the way in which indeterminacy is articulated in natural language; so we should go on taking lessons from natural language. And if we do this, we can see that the fact that there is something which is both p_1 and p_2 should be articulated as $(p_1\&p_2)(\boldsymbol{\varepsilon})$. This would, of course, require us to enrich $L^{\varepsilon,\leftarrow}$ with the possibility of forming complex predicates, which might seem to take us far beyond the boundaries of first-order predicate calculus. However, this is not true: as Quine (1996) points out, to do predicate logic in terms of what he calls *predicate functors* is a natural (albeit unusual) possibility[5].

Now what about universal quantification? There are again several ways to introduce it into $L^{\varepsilon,\leftarrow}$. The first is to make it simply dual to the existential quantification, i.e. to introduce a universal term $\boldsymbol{\pi}$ functioning in such a way that for every predicate p it holds that (where ~ is the predicate-functor of negation):

[5] By this reformulation of the predicate calculus we effectively get rid of variables, which is a good way to make logic closer to natural language - as I argued elsewhere (see Peregrin, in press).

$p(\boldsymbol{\pi}) = \neg(\sim p)(\boldsymbol{\varepsilon})$

(Note that one of the consequences of introducing predicate functors is that every statement can be articulated into a subject-predicate form, so the definition may be seen as entirely general.) This definition, however, of course depends on the way we decide to define negation. If we define $\neg$ in the most straightforward way shown above (and if $\sim$ is taken to denote simply the set-theoretical complement), it is easy to see that

$$\begin{aligned}\|\neg(\sim p)(\boldsymbol{\varepsilon})\| = {} & \{\langle x,x\rangle \mid \text{there is no y so that } \langle x,y\rangle \in \|(\sim p)(\boldsymbol{\varepsilon})\|\} = \\ & \{\langle x,x\rangle \mid \text{there is no y so that } \langle x,y\rangle \in \|\boldsymbol{\varepsilon}\| \text{ and } y \in \|\sim p\|\} = \\ & \{\langle x,x\rangle \mid \text{there is no y so that } y \in \|\sim p\|\} = \\ & \{\langle x,x\rangle \mid \text{for every y, } y \in \|p\|\}\end{aligned}$$

Another possibility would be to accept the ordinary general quantification of predicate logic. This would amount to the (re)introduction of the whole machinery of variables and binding of predicate logic, and it would make our logic into ordinary logic with the dynamic machinery *besides* the ordinary, static one. (Of course we then could define classical existential quantification in terms of the universal one, and dynamic universal quantification in terms of the dynamic existential one; so we would have both full sets of quantifiers.)

A third possibility would be to have no explicit universal quantification at all and to analyze corresponding locutions of natural language in terms of dynamic implication. (In contrast to negation, implication as defined above is *not* merely a straightforward transposition of the *material* implication of static logic; and this is what makes it possible to use it for the purposes of analyzing universal statements). Thus we could analyze

Every human is mortal

as

$human(\boldsymbol{\varepsilon}) \rightarrow mortal(\leftarrow)$

which may be read roughly as *If something is human, then it is mortal*. It is again easy to see that

$$\begin{aligned}& \|human(\boldsymbol{\varepsilon}) \rightarrow mortal(\leftarrow)\| = \\ & \{\langle x,x\rangle \mid \text{for every } \langle x,y\rangle \in \|human(\boldsymbol{\varepsilon})\| \text{ there is a z so that } \langle y,z\rangle \in \|mortal(\leftarrow)\|\} = \\ & \{\langle x,x\rangle \mid \text{for every } \langle x,y\rangle \text{ such that } \langle x,y\rangle \in \|\boldsymbol{\varepsilon}\| \text{ and } y \in \|human\| \text{ there is a z so that} \\ & \qquad \langle y,z\rangle \in \|\leftarrow\| \text{ and } z \in \|mortal\|\} = \\ & \{\langle x,x\rangle \mid \text{for every } \langle x,y\rangle \text{ such that } y \in \|human\| \text{ there is a z so that } y = z \text{ and} \\ & \qquad z \in \|mortal\|\} = \\ & \{\langle x,x\rangle \mid \text{for every } \langle x,y\rangle \text{ such that } y \in \|human\| \text{ it holds that } y \in \|mortal\|\} = \\ & \{\langle x,x\rangle \mid \text{for every y such that } y \in \|human\| \text{ it holds that } y \in \|mortal\|\} = \\ & \{\langle x,x\rangle \mid \|human\| \subseteq \|mortal\|\}\end{aligned}$$

This restriction to ‘unselective’ universal quantification is known, e.g., from DRT.

8. Concluding Remarks

The basic trick which allows for the logical accommodation of anaphoric effects is taking sentences as denoting not sets of indices, but rather relations between sets of indices. The more fine-grained structure we then give to the indices, the richer repertoire of anaphoric effects we can have. Within $L^{\varepsilon,\leftarrow}$, we took the indices to be, in effect, simply individual elements of the universe, and thus we were able to have only one, 'unselective' 'backward-looking term'. Within $L^{\varepsilon,\leftarrow,n}$ we improved on the situation by taking the indices to be n-tuples of elements of the universe: this allowed us to have n distinct, selective 'backward-looking terms'. Within $L^{the,a}$, the structure of the indices was still richer: they were taken to be choice functions; and as the consequence, we gained an infinite number of distinct backward-looking terms[6]. One can imagine also various further generalizations.

Now like in the case of introducing 'modal indices' (possible worlds), this introduction of 'dynamic indices' (underlying information states) can (and indeed, as I am convinced, *should*) be interpreted neither as a metaphysical, nor as a psychological, but rather as a purely logical achievement. We did not model any 'storage facilities' which we would find within human mind/brain; what we have done was to identify certain expressions with certain interesting inferential behavior within our language (*he, she, a man, the woman*, etc.), we have devised (rudimentary) logical systems in which it is possible to reflect the workings of such expressions (i.e. the inferential patterns governing their usage), and we have devised some simple model theories for these logics.

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[6] A different possibility is to structure the indices by means of some *ad hoc* linguistic items like the *discourse markers* of Groenenedijk's & Stokhof's *DPL*. See Peregrin and von Heusinger, 1997, for an indication of why this might be considered as unsatisfactory.

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Appendix: **the Calculi**

The language **L**

Vocabulary:
unary predicate constants (pc's), individual constants (ic's), **&**

Grammar:
if p is a pc and i an ic then $p(i)$ is a statement (st)
if s_1 and s_2 are st's, then s_1 **&** s_2 is a st.

Standard semantics:
if i is an ic, then $\|i\| \in U$ (where U is the universe)
if p is a pc, then $\|p\| \subseteq U$
if s is a st, then $\|s\|$ is a truth value; s is *true* iff $\|s\|$ = T, it is *false* iff $\|s\|$ = F
$\|p(i)\|$ = T iff $\|i\| \in \|p\|$
$\|s_1 \mathbin{\&} s_2\|$ = T iff $\|s_1\|$= T and $\|s_2\|$ = T

Alternative semantics 1:
if i is an ic, then $\|i\|$ is a total constant function from U into U
if p is a pc, then $\|p\| \subseteq U$
if s is a st, then $\|s\|$ is a function from U into U; s is *true* iff $\|s\|$ is total, it is *false* iff $\|s\|$ is empty
$\|p(i)\| = \{\langle x,y\rangle \mid \langle x,y\rangle \in \|i\| \text{ and } y \in \|p\|\}$ (thus: if the constant value of $\|i\|$ is an element of $\|p\|$, then $\|p(i)\| = \|i\|$, else $\|p(i)\| = \varnothing$)
$\|s_1 \mathbin{\&} s_2\| = \|s_2\|(\|s_1\|)$

Alternative semantics 2:
if i is an ic, then $\|i\| \subseteq U$ and $\|i\|$ is a singleton
if p is a pc, then $\|p\| \subseteq U$
if s is a st, then $\|s\|$ is a truth value; s is *true* iff $\|s\|$ = T, it is *false* iff $\|s\|$ = F
$\|p(i)\|$ = T iff $\|p\| \cap \|i\| \neq \varnothing$
$\|s_1 \mathbin{\&} s_2\|$ = T iff $\|s_1\|$ = T and $\|s_2\|$ = T

Alternative semantics 3:
if i is an ic, then $\|i\| \subseteq UxU$, such that $\|i\| = \{\langle x,x_i\rangle \mid x \in U\}$ for some $x_i \in U$
if p is a pc, then $\|p\| \subseteq U$
if s is a st, then $\|s\| \subseteq UxU$; s is *true* iff for every $x \in U$ there exists an y such that $\langle x,y\rangle \in \|s\|$, it is *false* iff for no $x \in U$ there exists an y such that $\langle x,y\rangle \in \|s\|$
$\|p(i)\| = \{\langle x,y\rangle \mid \langle x,y\rangle \in \|i\| \text{ and } y \in \|p\|\}$
$\|s_1 \mathbin{\&} s_2\| = \{\langle x,y\rangle \mid \text{there is a z such that } \langle x,z\rangle \in \|s_1\| \text{ and } \langle z,y\rangle \in \|s_2\|\}$

The language $\mathbf{L}^{\varepsilon,\leftarrow}$

Vocabulary:
pc's; terms (t's); **&**
t's are ic's, $\boldsymbol{\leftarrow}$**,** $\boldsymbol{\varepsilon}$

Grammar:
if p is a pc and t a t, then $p(t)$ is a st
if s_1 and s_2 are st's, then s_1 **&** s_2 is a st

Semantics:
if t is a t, then $\|t\| \subseteq U x U$; $\|\boldsymbol{\varepsilon}\| = UxU$, $\|\boldsymbol{\leftarrow}\| = \{<x,x> \mid x \in U\}$; for every ic i there exists a $x_i \in U$ such that $\|i\| = \{<x,x_i> \mid x \in U\}$
if p is a pc, then $\|p\| \subseteq U$
if s is a st, then $\|s\| \subseteq UxU$; s is *true* iff for every $x \in U$ there exists an y such that $<x,y> \in \|s\|$, it is *false* iff for no $x \in U$ there exists an y such that $<x,y> \in \|s\|$
$\|p(i)\| = \{<x,y> \mid <x,y> \in \|i\| \text{ and } y \in \|p\|\}$
$\|s_1 \mathbin{\&} s_2\| = \{<x,y> \mid \text{there is a z such that } <x,z> \in \|s_1\| \text{ and } <z,y> \in \|s_2\|\}$

The language $\mathbf{L}^{\boldsymbol{\varepsilon},\leftarrow,\mathbf{n}}$

Vocabulary:
pc's; terms of category k (tk's) for k=1,...,n; **&**
tk's are individual constants of category k (ick's), $\boldsymbol{\leftarrow}_{\mathbf{k}}$, $\boldsymbol{\varepsilon}_{\mathbf{k}}$

Grammar:
if p is a pc and t a t, then $p(t)$ is a st
if s_1 and s_2 are st's, then s_1 **&** s_2 is a st

Semantics:
if t is a tk, then $\|t\|\subseteq U^n x U^n$; $\|\boldsymbol{\varepsilon}_{\mathbf{k}}\| = \{<[x]_n,[y]_n,> \mid [y]_n^j=[x]_n^j \text{ for } j\neq k\}$, $\|\boldsymbol{\leftarrow}_{\mathbf{k}}\| = \{<[x]_n,[y]_n,> \mid [y]_n=[x]_n\}$; for every ick i there exists an $x_i\in U$ such that $\|i\|=\{<[x]_n,[y]_n,> \mid [y]_n^k=x_i \text{ and } [y]_n^j=[x]_n^j \text{ for } j\neq k\}$
if p is a pc, then $\|p\|\subseteq U$
if s is a st, then $\|s\|\subseteq U^n x U^n$; s is *true* iff for every $[x]_n\in U^n$ there exists an $[y]_n$ such that $<[x]_n,[y]_n>\in\|s\|$, it is *false* iff for no $[x]_n\in U^n$ there exists an y such that $<[x]_n,[y]_n>\in\|s\|$
$\|p(t_k)\| = \{<[x]_n,[y]_n > \mid <[x]_n,[y]_n >\in\|t_k\| \text{ and } [y]_n^k \in \|p\|\}$
$\|s_1 \,\&\, s_2\| =\{<[x]_n,[y]_n > \mid \text{there is a } [z]_n \text{ such that } < [x]_n,[z]_n >\in\|s_1\| \text{ and } < [z]_n,[y]_n > \in\|s_2\|\}$

The language $\mathbf{L}^{\mathbf{the,\,a}}$

Vocabulary:
pc's; ic's; **a**, **the**; **&**

Grammar:
ic is a t
if p is a pc, then **a**(p) and **the**(p) are t's
if p is a pc and t a t, then $p(t)$ is a st
if s_1 and s_2 are st's, then s_1 **&** s_2 is a st

Semantics:
Let $CHF_U = \{C \mid C$ is a partial function from Pow(U) to U such that if s belongs to the domain of C, then $C(s)\in s\}$.
if t is a t, then, $\|t\|\subseteq CHF_U x CHF_U$; for every ic i, $\|i\| = \{<C,C> \mid C\in CHF_U\}$
if t is a t and $C\in CHF_U$, then $|t|_C\in U$; for every ic i there exists a $x_i\in U$ such that $|i|_C = x_i$ for every $C\in CHF_U$
if p is a pc, then $\|p\|\subseteq U$
if s is a st, then $\|s\|\subseteq CHF_U x CHF_U$; s is *true* iff for every C there exists a C' such that $<C,C'>\in \|s\|$, it is *false* iff for no C there exists a C' such that $<C,C'>\in \|s\|$
$\|\mathbf{a}(p)\| = \{<C,C'> \mid C(s) = C'(s)$ for every $s \neq \|p\|$ and $C'(\|p\|) \in \|p\|\}$
$\|\mathbf{the}(p)\| = \{<C,C> \mid C(\|p\|) \in \|p\| \}$
$|\mathbf{a}(p)|_C = |\mathbf{the}(p)|_C = C(\|p\|)$
$\|p(t)\| = \{<C,C'> \mid <C,C'>\in \|t\|$ and $|t|_{C'}\in \|p\| \}$
$\|s_1 \mathbin{\&} s_2\| = \{<C,C'> \mid$ there is a C'' such that $<C,C''>\in \|s_1\|$ and $<C'',C'>\in \|s_2\|\}$